

1.1 Arithmetic Sequences – NOTES

A **sequence** is AN ORDERED LIST OF OBJECTS.
 It contains terms that FOLLOW A PATTERN or rule to determine the next term in the sequence.

One well-known sequence in mathematics is the **Fibonacci sequence**: 1, 1, 2, 3, 5, 8, 13...

What is the rule used to determine the next term in the Fibonacci sequence?

EACH # IS THE SUM OF THE TWO PRECEDING NUMBERS

An **arithmetic sequence** is based on an ADDITION PATTERN. It is an ordered list of objects in which THE DIFFERENCE BETWEEN THE

CONSECUTIVE TERMS IS CONSTANT. This constant is called the COMMON DIFFERENCE (d).

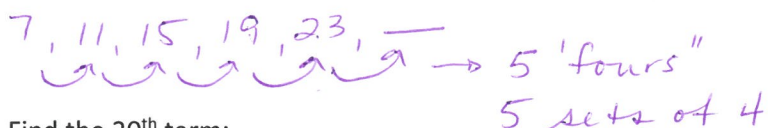
Example #1: For the sequence 7, 11, 15, 19, 23...

* PICK ANY CONSECUTIVE TERMS & SUBTRACT THEM

a) State the first term and common difference:

t_1 (or a) = 7
 $d = 11 - 7 = 4$
 $d = 15 - 11 = 4$

b) Find the 6th term:



START & ADD 5 SETS OF 4
 $t_6 = 7 + 5(4)$
 $= 7 + 20$
 $= 27$

c) Find the 20th term:

$t_{20} = 7 + 19(4) = 7 + 76 = 83$

↑ ONE LESS THAN THE TERM YOU ARE LOOKING FOR!

d) Find the 64th term:

$t_{64} = 7 + 63(4)$
 $= 7 + 252 = 259$

<p>In general,</p>	$t_n = t_1 + (n - 1)d$ ↑ SOMETIMES CALLED 'a'	<p>t_n = term in the nth position (general term) t_1 = a (first term) d = common difference n = term number</p>
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$$\begin{aligned} t_1 &= 20 \\ d &= -6 \end{aligned}$$

Example #2: For the sequence 20, 14, 8, 2...

a) Find t_{27} :

$$\begin{aligned} t_{27} &= 20 + (27-1)(-6) \\ &= 20 + 26(-6) \\ &= 20 + (-156) \\ &= -136 \end{aligned}$$

b) Find t_{18} :

$$\begin{aligned} t_{18} &= 20 + (18-1)(-6) \\ &= 20 + (17)(-6) \\ &= 20 + (-102) \\ &= -82 \end{aligned}$$

c) Write the expression for t_n (GENERAL TERM):

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 20 + (n-1)(-6) \\ &= 20 + (-6n) + 6 \end{aligned}$$

$$t_n = -6n + 26 \quad \leftarrow \text{LOOK FAMILIAR?}$$

$$y = mx + b$$

Example #3: Given the arithmetic sequence 17, 12, 7...

a) Determine the general term.

$$\begin{aligned} t_n &= 17 + (n-1)(-5) \\ &= 17 + (-5n) + 5 \\ t_1 &= 17 \\ d &= -5 \end{aligned}$$

$$t_n = -5n + 22$$

b) Use the general term to determine t_{23} :

$$\begin{aligned} n &= 23 \\ t_{23} &= -5(23) + 22 \\ &= -115 + 22 \\ &= -93 \end{aligned}$$

Example #4: For the following arithmetic sequences, determine the missing terms.

a) $-11, -2, 7, 16, 25$
 (1) $d = 25 - 16 = 9$
 (2) SUBTRACT BACKWARDS

b) $6, 8, 10, 12, 14$
 (1) $t_n = t_1 + (n-1)d$
 $14 = 8 + (4-1)d$
 $14 = 8 + 3d$
 $-8 \quad -8$
 $\frac{6}{3} = \frac{3d}{3} \Rightarrow 2 = d$
 (2) SUBTRACT BACKWARDS
 $t_1 = 8$
 $t_4 = 14$
 $n = 4$
 *USE ALGEBRA

c) If 27 is the first term and 75 is the 5th term, find the terms in between.

$t_1 = 27$
 $t_5 = 75$
 $n = 5$
 (1) $75 = 27 + (5-1)d$
 $75 = 27 + 4d$
 $-27 \quad -27$
 $\frac{48}{4} = \frac{4d}{4}$
 $12 = d$
 (2) $27, 39, 51, 63, 75$
 $+12 \quad +12 \quad +12$

d) If $t_9 = 76$ and $t_{15} = 118$, find the common difference and the first term.

(1) FIND COMMON DIFFERENCE:
 $118 = 76 + (7-1)d$
 $118 = 76 + 6d$
 $-76 \quad -76$
 $\frac{42}{6} = \frac{6d}{6}$
 $7 = d$

(2) SUBTRACT BACK FROM t_9
 OR
 USE FORMULA:
 $t_n = t_1 + (n-1)d$

$76 = t_1 + (9-1)7$
 $76 = t_1 + (8)7$
 $76 = t_1 + 56$
 $-56 \quad -56$
 $20 = t_1$

$t_9 = 76$
 $n = 9$
 $d = 7$

