1.2 Arithmetic Series

Arithmetic Series: The sum of the terms of an arithmentic sequence.

Sequence: 3, 7, 11, 15, 19,...
Series: 3 + 7 + 11 + 15 + 19 + ...

Investigate: Find the sum of 1 + 2 + 3 + 4 +... +97+ 98 + 99 + 100

101

101

101

$$\frac{100}{2}(101) = 50(101) = 5050$$

Ex. Find the sum of the first 20 terms of 7 + 11 + 15 + ...

$a = 7$
$d = 4$
$n = 20$

① $t_{20} = a + (n-1)d$
$= 7 + 19(4)$
$= 7 + 76$
$= 83$

② $S_{20} = \frac{20}{2}(7 + 83)$
1st term
last term
$= 10(90)$
$= 900$

In general, the sum of the first *n* terms is...

$$S_n = \frac{n}{2}(a + t_n)$$

a = first term
t_n = last term
n = number of terms

Ex. Given 17 + 11 + 5 + -1 +.... Find S_{30}

a = 17
n = 30
t_{30} =
d = −6

① $t_{30} = \boxed{a + (n-1)d}$
$= 17 + 29(-6)$
$= 17 - 174$
$= -157$

$a + a + (n-1)d$

② $S_{30} = \frac{30}{2}[17 + (-157)]$
$= 15(-140)$
$= -2100$

Note: we could also use the formula $\boxed{S_n = {}^n/_2\,[2a + d(n-1)]}$ -Use if we know: n, a, d

$$S_n = \frac{n}{2}(a + t_n)$$
$$= \frac{n}{2}[a + a + (n-1)d]$$
$$= \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2}[2a + d(n-1)]$$

Ex. Find the sum of 7 + 14 + 21 + ... +266

$a = 7$
$n = ?$
$t_n = 266$
$d = 7$

① $266 = 7 + (n-1)7$

$266 = 7 + 7n - 7$

$\frac{266}{7} = \frac{7n}{7}$

$38 = n$

② $S_{38} = \frac{38}{2}(7 + 266)$

$= 19(273)$

$= 5187$

Ex. If $a = -6$, $t_n = 21$ and $S_n = 75$, determine n.

$S_n = \frac{n}{2}(a + t_n)$

$75 = \frac{n}{2}(-6 + 21)$

$2 \cdot 75 = \frac{n}{\cancel{2}}(15) \cdot \cancel{2}$

$\frac{75}{15} = \frac{15n}{15} \Rightarrow n = 10$

Ex. Dominoes are displayed in an arithmetic sequence. The first row has 4 dominoes, the last row has 106 dominoes, and there are 1925 dominoes in total. Find the number of rows of dominoes.

$a = 4$
$t_n = 106$
$S_n = 1925$

$1925 = \frac{n}{2}(4 + 106)$

$2 \cdot 1925 = \frac{n}{\cancel{2}}(110) \cdot \cancel{2}$

$\frac{3850}{110} = \frac{110n}{110} \Rightarrow n = 35$

THERE ARE 35 ROWS.

Ex. A pile of bricks is arranged in rows. The number of bricks in each row forms the arithmetic sequence 65, 59, 53, ... How many bricks are there if there are 9 rows?

$a = 65$
$d = -6$
$n = 9$
$t_n = ?$

① $t_n = 65 + (9-1)(-6)$

$t_n = 65 + 8(-6)$

$t_n = 65 + (-48)$

$t_n = 17$

② $S_n = \frac{n}{2}(a + t_n)$

$= \frac{9}{2}(65 + 17)$

$= 4.5(82)$

$= 369$

OR DO IT ALL IN ONE STEP USING: $S_n = \frac{n}{2}[2a + d(n-1)]$

$= \frac{9}{2}[2(65) + (-6)(9-1)]$

$= 4.5[130 + (-6)(8)]$

$= 4.5[130 + (-48)]$

$= 4.5(82)$

$= 369$

Ex. Determine the first term if $d = -6$, $S_n = 32$, and $n = 13$

$a = ?$
$d = -6$
$n = 13$
$S_n = 32$

$S_n = \frac{n}{2}[2a + d(n-1)]$

$32 = \frac{13}{2}[2a + (-6)(12)]$

$32 = 6.5(2a - 72)$

$32 = 13a - 468$

$+468 \qquad\qquad +468$

$\frac{500}{13} = \frac{13a}{13} \Rightarrow a \doteq 38.5$