4.1 Perfect Squares, Perfect Cubes, and Their Roots

A **perfect square** is a whole number that can be represented as

a product of two equal factors.

Rectangle
$A = lw$

Square
$A = s^2$

Area = 5^2

$A = 25$

5

Area = 64

$s = 8$

The side length of the square is the square root of the area.
($\sqrt{}$ means positive #)

$\sqrt{64} = 8$

Perfect Squares: can be represented as a square with that area (square root is its side length).

Examples:

a) $7 \times 7 =$ 49

b) $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

c) $(0.3)^2 =$ 0.09

d) $(-3)(-3) = (-3)^2 = 9$ * squares are always positive.

Finding a Square Root

Method 1 – Prime Factors

$\sqrt{1764}$

2 | 1764
2 | 882
3 | 441
3 | 147
7 | 49
7

* break in to two sets!

$= (2 \cdot 3 \cdot 7) \cdot (2 \cdot 3 \cdot 7)$

$= 42 \cdot 42$

So $\sqrt{1764} =$ 42

Practice:

a) $\sqrt{576}$

$$\begin{array}{r|l} 2 & 576 \\ 2 & 288 \\ 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

$(2\cdot2\cdot2\cdot3)\cdot(2\cdot2\cdot2\cdot3)$

$= 24\cdot24$

$\sqrt{576} = 24$

b) $\sqrt{256}$

$$\begin{array}{r|l} 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

$(2\cdot2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2)$

$16\cdot16$

$\sqrt{256} = 16$

Method 2 – Estimate and Check

$\sqrt{1296}$

$30^2 = 900$ and $40^2 = 1600$

$900 < 1296 < 1600$

so $30^2 < \sqrt{1296} < 40^2$

(1296 is about halfway between 30^2 and 40^2)

Try 35: $35^2 = 1225$

Try 36: $36^2 = 1296$

So $\sqrt{1296} = 36$

Method 3 – Calculator

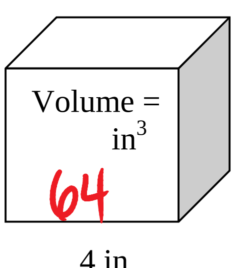
$\sqrt{441} = 21$

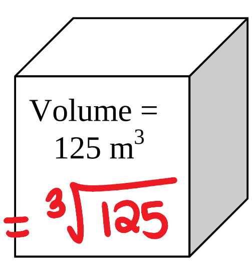
A **perfect cube** is a whole number that can be represented as

a product of 3 equal factors.

Rectangular *Prism*
$V = lwh$

Cube
$V = s^3$





$s = \sqrt[3]{125}$

The side length of the cube is the <u>cube root</u> of the area.

$\sqrt[3]{125} = 5$

Perfect Cubes: can be represented as a cube with that volume (cube root is side length)

Finding a Cube Root

- look for 3 sets

Method 1 – Prime Factors

$\sqrt[3]{2744} =$

$$\begin{array}{r|l} 2 & 2744 \\ 2 & 1372 \\ 2 & 686 \\ 7 & 343 \\ 7 & 49 \\ & 7 \end{array}$$

$= (2 \cdot 7) \cdot (2 \cdot 7) \cdot (2 \cdot 7)$

$= 14 \cdot 14 \cdot 14$

$\sqrt[3]{2744} = 14$

Practice:

a) $\sqrt[3]{1331}$

$$\begin{array}{r|l} 11 & 1331 \\ 11 & 121 \\ & 11 \end{array}$$

$11 \cdot 11 \cdot 11$

$\sqrt[3]{1331} = 11$

b) $\sqrt[3]{729}$

$$\begin{array}{r|l} 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

$= (3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3)$

$= 9 \cdot 9 \cdot 9$

$\sqrt[3]{729} = 9$

Method 2 – Estimate and Check

$\sqrt[3]{1728} =$

$10^3 =$ 1000 and $20^3 =$ 8000

1000 $<$ 1728 $<$ 8000

so $10^3 < \sqrt[3]{1728} < 20^3$

(1728 is closer to 1000)

Try 11: $11^3 =$ 1331

Try 12: $12^3 =$ 1728

So $\sqrt[3]{1728} =$ 12

Method 3 – Calculator

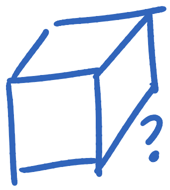
maybe 2nd or shift

$3 * \sqrt{}$ (4913)

$\sqrt[3]{4913} = 17$

Example:

A cube has a volume of 2197 cm^3. What is its side length?



$\sqrt[3]{2197} = 13$ cm

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