Date: _____

# 4.2 Estimating Roots and Irrational Numbers

## A. Rational vs. Irrational Numbers

**Real Numbers**

**Rational Numbers** (fraction, terminating or repeating decimal)
-0.333, 0.625, 2/3, 1 1/2

**Integers**
... -3, -2, -1, 0, 1, 2, 3...

**Whole Numbers**
0, 1, 2, 3...

**Natural Numbers**
1, 2, 3...

**Irrational**
cannot be written in the form $\frac{m}{n}$ where m and n are integers and $n \neq 0$.
1.73205..., $\sqrt{30}$, $\sqrt{\frac{5}{9}}$, $\pi$, $-\sqrt{2}$

**Example 1: Rational or Irrational? Explain.**

a) $\sqrt{0.09}$

9 is a perfect square and the decimal has moved 2 spaces.

$\sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$ ✓

rational

b) $\sqrt{0.016}$

16 is a perfect square and the decimal place has moved 3 spaces

$\sqrt{\frac{16}{1000}} = \frac{4}{\times}$

irrational

c) $\sqrt{\frac{49}{64}}$

$\frac{\sqrt{49}}{\sqrt{64}} = \frac{7}{8} = 0.875$ rational

d) $\sqrt[3]{\frac{27}{100}}$

$\sqrt[3]{27} = 3$

irrational

$\sqrt[3]{100} = \times$

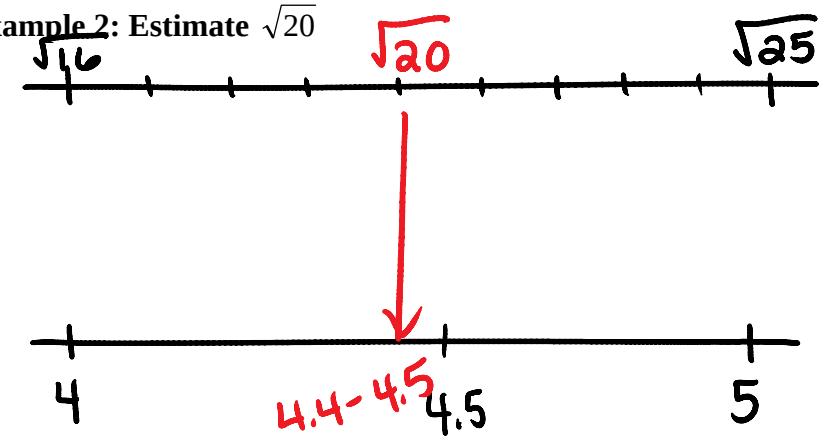
(4.64158...)

## B. Estimating Non-Perfect Squares and Cubes

1) Find benchmarks (grade 9)
2) Approximate answer
3) Guess & Check → this is the only step you can use your calculator

$\sqrt{16}$ ↖ $\sqrt{20}$ ↗ $\sqrt{25}$

**Example 2: Estimate $\sqrt{20}$**

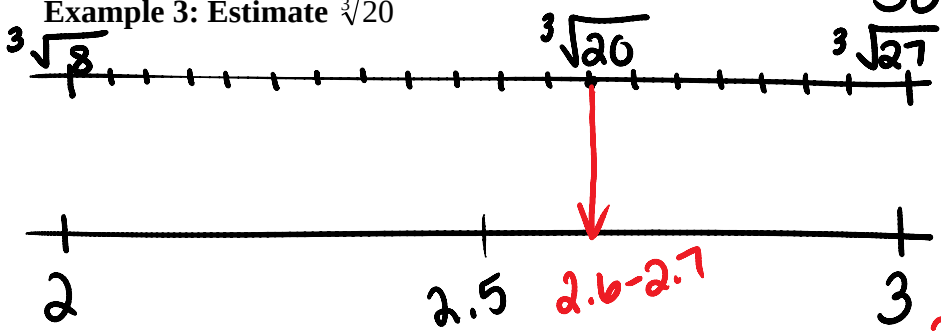



Between 4.4 and 4.5
Check.

$4.45^2 = 19.80$ to small
$4.47^2 = 19.98$ 0.02 diff.
$4.48^2 = 20.07$ 0.07 diff

So, $\sqrt{20} = 4.47$

**Example 3: Estimate $\sqrt[3]{20}$**




$2.71^3 = 19.903$ 0.097
$2.72^3 = 20.124$ 0.124

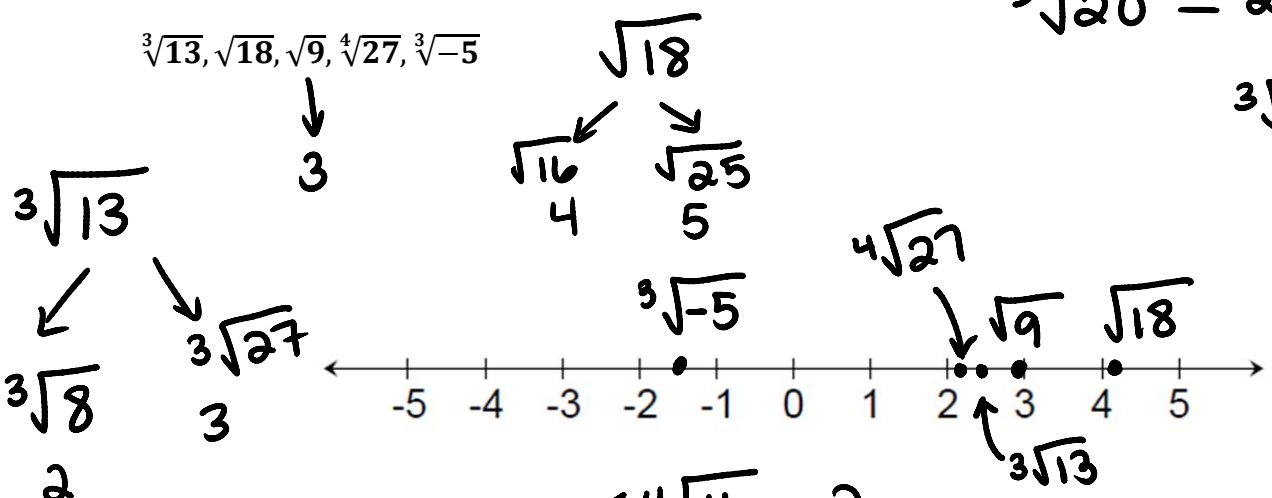
Between 2.6 and 2.7. Check

$2.6^3 = 17.576$ } too small
$2.7^3 = 19.683$ }

$\sqrt[3]{20} \simeq 2.71$

**Example 4: Use a number line to order from least to greatest**

$\sqrt[3]{13}, \sqrt{18}, \sqrt{9}, \sqrt[4]{27}, \sqrt[3]{-5}$




$\sqrt[3]{-1}$ -1
$\sqrt[3]{-5}$
$\sqrt[3]{-8}$ -2

$\sqrt[3]{13} = 2.35$

$\sqrt[4]{27}$ / $\sqrt[4]{16} = 2$

$\sqrt[4]{81} = 3$

$\sqrt[4]{27} = 2.27$

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