

4.3 Mixed and Entire Radicals

A. Entire → Mixed Radicals

Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9}$$

$$= 2 \cdot 3 = 6$$

$$\sqrt{36} = \sqrt{4 \cdot 9} = 6$$

This property can be used to simplify square and cube roots that are **not** perfect, but have factors that are perfect.

entire radical
 $\sqrt{24}$

The factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24
 perfect square

$$= \sqrt{4} \cdot \sqrt{6} = 2 \cdot \sqrt{6} = 2\sqrt{6}$$

mixed radical

Examples:

a) $\sqrt{80}$

$$= \sqrt{8 \cdot 10}$$

* look for factors that appear twice

$$= \sqrt{2 \cdot 2 \cdot 2 \cdot 10} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$$

$$= \sqrt{2 \cdot 2} \cdot \sqrt{2 \cdot 2} \cdot \sqrt{5} = 2 \cdot 2 \cdot \sqrt{5} = 4\sqrt{5}$$

d) $\sqrt[3]{144}$

$$\sqrt[3]{12 \cdot 12}$$

$$\sqrt[3]{2 \cdot 2 \cdot 3 \cdot 2 \cdot 2 \cdot 3}$$

$$2^3 \sqrt[3]{3 \cdot 2 \cdot 3}$$

$$2^3 \sqrt[3]{18}$$

b) $\sqrt{72}$

$$= \sqrt{8 \cdot 9} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$$

$$= 2 \cdot 3 \sqrt{2}$$

$$= 6\sqrt{2}$$

e) $\sqrt[4]{32}$

$$\sqrt[4]{16 \cdot 2}$$

$$\sqrt[4]{8 \cdot 2 \cdot 2} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$

$$2^4 \sqrt[4]{2}$$

c) $\sqrt[3]{128}$

$$= \sqrt[3]{4 \cdot 32}$$

$$= \sqrt[3]{2 \cdot 2 \cdot 8 \cdot 4}$$

$$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$

$$= 2 \cdot 2^3 \sqrt[3]{2} = 4^3 \sqrt[3]{2}$$

f) $\sqrt[4]{162}$

$$\sqrt[4]{81 \cdot 2}$$

$$= \sqrt[4]{9 \cdot 9 \cdot 2}$$

$$= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 2}$$

$$= 3^4 \sqrt[4]{2}$$

$$\begin{aligned}
 \text{g) } \sqrt{x^4} &= \sqrt{x^2 \cdot x^2} \\
 &= \sqrt{\cancel{x \cdot x} \cdot \cancel{x \cdot x}} \\
 &= x \cdot x = x^2
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } \sqrt[3]{16x^2y^4} \\
 &= \sqrt[3]{\cancel{2 \cdot 2 \cdot 2} \cdot 2 \cdot x \cdot x \cdot \cancel{y \cdot y \cdot y \cdot y}} \\
 &= 2y^3 \sqrt{2x^2y}
 \end{aligned}$$

B. Mixed → Entire Radicals

Examples:

a) $4\sqrt{3}$

$$\begin{aligned}
 &\sqrt{4 \cdot 4 \cdot 3} \\
 &= \sqrt{48}
 \end{aligned}$$

d) $3\sqrt[3]{2}$ *← 3 times*

$$\begin{aligned}
 &3\sqrt[3]{3 \cdot 3 \cdot 3 \cdot 2} \\
 &= 3\sqrt[3]{27 \cdot 2} = 3\sqrt[3]{54}
 \end{aligned}$$

b) $2\sqrt{7}$

$$\begin{aligned}
 &\sqrt{2 \cdot 2 \cdot 7} \\
 &= \sqrt{4 \cdot 7} = \sqrt{28}
 \end{aligned}$$

e) $2\sqrt[3]{12}$

$$\begin{aligned}
 &2\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 12} \\
 &= 2\sqrt[3]{8 \cdot 12} = 2\sqrt[3]{96}
 \end{aligned}$$

c) $3\sqrt{10}$

$$\begin{aligned}
 &\sqrt{3 \cdot 3 \cdot 10} \\
 &= \sqrt{9 \cdot 10} = \sqrt{90}
 \end{aligned}$$

f) $2\sqrt[5]{2}$ *← 5 times*

$$\begin{aligned}
 &2\sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \\
 &= 2\sqrt[5]{32 \cdot 2} \\
 &= 2\sqrt[5]{64}
 \end{aligned}$$