4.4 Fractional Exponents and Radicals – Part 1

Recall **product of powers** exponent law:

$$a^m \cdot a^n = a^{m+n}$$

This can be extended to powers with fractional exponents (with 1 as the numerator).

$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} = 5^{\frac{2}{2}} = 5^1$

$\sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{25} = 5$

$5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent

$\boxed{a^{\frac{1}{2}} = \sqrt{a}}$

$5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5^{\frac{3}{3}} = 5^1$

$\sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5} = \sqrt[3]{5 \cdot 5 \cdot 5} = \sqrt[3]{125}$
$= 5$

$5^{\frac{1}{3}}$ and $\sqrt[3]{5}$ are equivalent

$\boxed{a^{\frac{1}{3}} = \sqrt[3]{a}}$

So, $\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$

Examples: ***Write each power as a radical and evaluate.***

a) $27^{\frac{1}{3}}$ $= \sqrt[3]{27}$
$= 3$

b) $100^{0.5}$ $= 100^{\frac{1}{2}}$
$= \sqrt{100}$
$= 10$

c) $(-64)^{\frac{1}{3}}$ $= \sqrt[3]{-64}$
$= -4$

d) $256^{\frac{1}{4}}$ $= \sqrt[4]{256}$
$= 4$

What if the numerator is not a 1?

Recall **power of a power** exponent law:

$$\left(a^m\right)^n = a^{mn} \qquad (8^2)^3 = 8^6$$

We can use this exponent law when the numerator is not a 1.

$8^{\frac{2}{3}}$ $\qquad \frac{2}{3}$ can be written as $\frac{1}{3}\cdot 2$ or $2\cdot\frac{1}{3}$

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$$

* brackets are important!

or $\quad 8^{\frac{2}{3}} = \left(8^2\right)^{1/3} = \sqrt[3]{64} = 4$

$$\boxed{a^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m} \text{ or } \sqrt[n]{x^m}$$

Examples: ***Write each power as a radical and evaluate.***

a) $16^{\frac{3}{2}}$ $= \left(\sqrt{16}\right)^3$

$= (4)^3 = 64$

c) $27^{\frac{2}{3}}$

$\left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

b) $81^{\frac{3}{4}}$

$\left(\sqrt[4]{81}\right)^3$

$= (3)^3 = 27$

d) $(-32)^{0.4}$ $\quad 0.4 \quad \frac{4}{10} = \frac{2}{5}$

$(-32)^{2/5}$

$= \left(\sqrt[5]{-32}\right)^2 = (-2)^2 = 4$

page 227 #3-7 all