

4.4 Fractional Exponents and Radicals – Part 2

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

DENOMINATOR \leftrightarrow INDEX

Examples: Write each radical as a power.

a) $\sqrt[2]{3^5} = 3^{\frac{5}{2}}$

c) $\sqrt[3]{19^4} = 19^{\frac{4}{3}}$

b) $\sqrt[3]{25^2} = 25^{\frac{2}{3}}$

d) $\sqrt{\left(\frac{2}{5}\right)^7} = \left(\frac{2}{5}\right)^{\frac{7}{2}}$

Examples: Write each power as a radical and evaluate.

a) $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\left(\frac{4}{9}\right)^1} = \sqrt{\frac{4}{9}}$
 power radical
 $= \frac{\sqrt{4}}{\sqrt{9}}$
 $= \frac{2}{3}$

d) $16^{0.75} = 16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3$
 $\frac{75}{100} \div 25 = \frac{3}{4}$
 $= 2^3$
 $= 8$

b) $0.49^{\frac{1}{2}} = \sqrt{0.49}$
 $= 0.7$
 Split decimal place in half

e) $\left(\frac{4}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{25}}\right)^3$
 $= \left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$
 $= \frac{8}{125}$

c) $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{16}{81}}$
 $= \sqrt[4]{16}$
 $= \sqrt[4]{81}$
 $= \frac{2}{3}$

f) $0.04^{\frac{3}{2}} = \left(\sqrt{0.04}\right)^3$
 $= (0.2)^3 = (0.2)(0.2)(0.2)$
 $= 0.008$

Example 3: Write as a power with exponent $\frac{1}{2}$ and then write as a radical.

$$\text{a) } 4 = \boxed{16}^{\frac{1}{2}} = \sqrt{\boxed{16}}$$

$$\text{b) } 5 = \boxed{25}^{\frac{1}{2}} = \sqrt{\boxed{25}}$$

Example 3: Write as a power with exponent $\frac{1}{3}$ and then write as a radical.

$$\text{a) } 2 = \boxed{8}^{\frac{1}{3}} = \sqrt[3]{\boxed{8}}$$

↑
↑
↑
standard
exponential
radical
form
form
form

$$\text{b) } -4 = \boxed{-64}^{\frac{1}{3}} = \sqrt[3]{\boxed{-64}}$$

Example 4:

Brain mass can be estimated with the formula:

$$b = 0.01m^{\frac{2}{3}}$$

where b is the brain mass (in kg) and m is the body mass (in kg).

Estimate the brain mass of each animal.

a) husky 27 kg

$$\begin{aligned}
 b &= 0.01(27)^{\frac{2}{3}} \\
 &= 0.01(\sqrt[3]{27})^2 \\
 &= 0.01(3)^2 \\
 &= 0.01(9) \\
 &= 0.09 \text{ kg}
 \end{aligned}$$

b) polar bear 200 kg

$$\begin{aligned}
 b &= 0.01(200)^{\frac{2}{3}} \\
 &= 0.01(\sqrt[3]{200})^2 \quad * \text{ USE CALC.} \\
 &= 0.01(34) \quad \leftarrow \text{rounded to whole \#} \\
 &= 0.34 \text{ kg}
 \end{aligned}$$