

4.5 Negative Exponents and Reciprocals

Complete the following table:

2^4	16	$\downarrow \div 2$ $\downarrow \div 2$ $\downarrow \div 2$
2^3	8	
2^2	4	
2^1	2	
2^0	1	
2^{-1}	$\frac{1}{2}$	$\leftarrow \cdot 2$ $\leftarrow \cdot 2$ $\leftarrow \cdot 2$ $\leftarrow \cdot 2$
2^{-2}	$\frac{1}{4}$	
2^{-3}	$\frac{1}{8}$	
2^{-4}	$\frac{1}{16}$	

CONT. TO FOLLOW THE PATTERN

The exponent laws still apply to negative exponents:

$$a^m \cdot a^n = a^{m+n}$$

$$5^{-2} \cdot 5^2 = 5^{-2+2} = 5^0 = 1$$

IF THE BASES ARE THE SAME, KEEP THE BASE & ADD THE EXPONENTS

ex) $2^4 \cdot 2^5 = 2^9$
 $\downarrow \quad \quad \quad \downarrow$
 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

** 5^{-2} & 5^2 ARE RECIPROCAL (THEY RESULT IN A ONE WHEN YOU MULT. THEM)

$$\frac{1}{5^2} \cdot 5^2 = 1$$

$$\frac{1}{25} \cdot 25 = \frac{25}{25} = 1$$

$$a^{-n} = \frac{1}{a^n} \quad \text{OR} \quad \frac{1}{a^{-n}} = a^n$$

"WRITE IN THE RECIPROCAL POSITION"

Example 1: Evaluate.

a) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

b) $\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$

c) $\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^3 = -\frac{64}{27}$ e) $0.3^{-4} = \left(\frac{3}{10}\right)^{-4} = \left(\frac{10}{3}\right)^4 = \frac{10000}{81}$

Diagram: A number line from -1 to 0 with a red dot at $-\frac{1}{3}$. Arrows point from the dot to the expressions above.

Annotations: "CHANGE TO FRACTION" (red arrow pointing to the fraction conversion in e), "RECIPROCAL" (red arrow pointing to the reciprocal conversion in c).

d) $\left(-\frac{2}{1}\right)^{-5} = \left(-\frac{1}{2}\right)^5 = -\frac{1}{32}$ f) $(-1.5)^{-2} = \left(-\frac{3}{2}\right)^{-2} = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

What if the exponent is a negative fraction???

- ① WRITE RECIPROCAL
- ② CHANGE TO A RADICAL

g) $8^{\frac{2}{3}} = \left(\frac{1}{8}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{1}{8}}\right)^2 = \left(\frac{\sqrt[3]{1}}{\sqrt[3]{8}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

h) $16^{\frac{5}{4}} = \left(\frac{1}{16}\right)^{\frac{5}{4}} = \left(\sqrt[4]{\frac{1}{16}}\right)^5 = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

i) $\left(\frac{25}{36}\right)^{\frac{1}{2}} = \left(\frac{36}{25}\right)^{\frac{1}{2}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$

j) $\left(\frac{9}{16}\right)^{\frac{3}{2}} = \left(\frac{16}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

Examples 2: Write as a power with negative exponents.

- ① FIND A + POWER
- ② CHANGE INTO - POWER BY WRITING RECIPROCAL

a) $8 = 2^3 = \frac{1}{2^{-3}}$

b) $\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

c) $\frac{9}{25} = \left(\frac{3}{5}\right)^2 = \left(\frac{5}{3}\right)^{-2}$

Example 3:

The speed at which dinosaurs travelled (from fossilized tracks) has been determined to be:

$$v = 0.155s^{\frac{5}{3}}f^{-\frac{7}{6}}$$

Calculate the speed (v) in m/s of a dinosaur with a foot length (f) of 0.25 m and 1.00 m between footprints (s).

$$\begin{aligned}v &= 0.155 \left(1.00\right)^{\frac{5}{3}} \left(0.25\right)^{-\frac{7}{6}} \\&= 0.155 (1) \left(0.25\right)^{-\frac{7}{6}} \\&= 0.155 \left(0.25\right)^{-\frac{7}{6}} \quad * \text{CALC.}\end{aligned}$$

$$\begin{aligned}" & 0.155 \times 0.25 \wedge \left(-\frac{7}{6}\right) = " \\& \doteq 0.78 \text{ m/s}\end{aligned}$$