Lesson 10: Factor $ax^2 + bx + c$ by Decomposition – Part 1

When $a \neq 1$ we need another method to factor trinomials called decomposition. In this method, the coefficient of the middle term is broken down (or "decomposed") into two smaller terms.

Steps: $2x^2 + 7x + 6$

1) Make sure the expression is in the correct order.

2) Find the product of $(a)(c)$. $(2)(6) = 12$

3) Find two numbers that sum to the middle term but have a product equal to $(a)(c)$.

$+3 \diagtimes^{7}_{12} +4$ $\begin{array}{l}1 \times 12 \\ 2 \times 6 \\ 3 \times 4\end{array}$

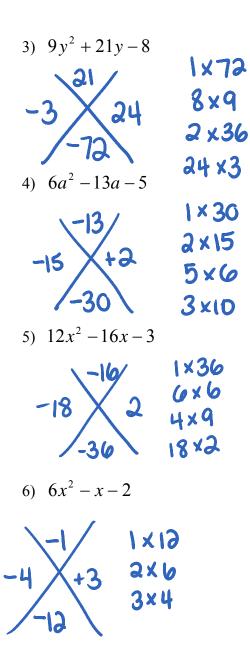4) Rewrite the expression splitting the middle term into the two terms determined above.

$$2x^2 + 4x + 3x + 6$$

5) Factor by grouping.

$$(2x^2 + 4x) + (3x + 6)$$
$$2x(x+2) + 3(x+2)$$

6) Write the factors.

$$(x+2)(2x+3)$$

Examples: *Factor.*

1) $6x^2 + 11x + 4$

$+8 \diagtimes^{11}_{24} +3$ $\begin{array}{l}1 \times 24 \\ 2 \times 12 \\ 8 \times 3 \\ 6 \times 4\end{array}$

$$(6x^2 + 3x) + (8x + 4)$$
$$3x(2x+1) + 4(2x+1)$$
$$(2x+1)(3x+4)$$

2) $4b^2 + 8b + 3$

$+6 \diagtimes^{8}_{12} +2$ $\begin{array}{l}1 \times 12 \\ 2 \times 6 \\ 3 \times 4\end{array}$

$$(4b^2 + 2b) + (6b + 3)$$
$$2b(2b+1) + 3(2b+1)$$
$$(2b+3)(2b+1)$$

3) $9y^2 + 21y - 8$

$$\begin{array}{c} 21 \\ -3 \diagtimes 24 \\ -72 \end{array}$$

1×72
8×9
2×36
24×3

$(9y^2 - 3y) + (24y - 8)$

$3y(3y - 1) + 8(3y - 1)$

$(3y - 1)(3y + 8)$

4) $6a^2 - 13a - 5$

$$\begin{array}{c} -13 \\ -15 \diagtimes +2 \\ -30 \end{array}$$

1×30
2×15
5×6
3×10

$(6a^2 + 2a) - (15a - 5)$

$2a(3a + 1) - 5(3a + 1)$

$(3a + 1)(2a - 5)$

5) $12x^2 - 16x - 3$

$$\begin{array}{c} -16 \\ -18 \diagtimes 2 \\ -36 \end{array}$$

1×36
6×6
4×9
18×2

$(12x^2 + 2x) - (18x - 3)$

$2x(6x + 1) - 3(6x + 1)$

$(2x - 3)(6x + 1)$

6) $6x^2 - x - 2$

$$\begin{array}{c} -1 \\ -4 \diagtimes +3 \\ -12 \end{array}$$

1×12
2×6
3×4

$(6x^2 + 3x) - (4x - 2)$

$3x(2x + 1) - 2(2x + 1)$

$(3x - 2)(2x + 1)$

page 177 #(12,13,15,16) aceg