

Lesson 12: Factoring Special Polynomials

Perfect Square Trinomials

A *perfect square trinomial* is formed from the product of two identical binomials. When you recognize a quadratic trinomial as a PST, it makes factoring it quite easy, since there is an inherent pattern.

Recall:

In order to be a PST: The first and last terms must be perfect squares, and the middle term must be twice the product of the square roots of the first and last terms.

$$(x + 3)^2$$

$$(x + 3)(x + 3)$$

$$x^2 + 6x + 9$$

$$(x - 3)^2$$

$$(x - 3)(x - 3)$$

$$x^2 - 6x + 9$$

The pattern when squaring a binomial: $(a + b)^2 = a^2 + 2ab + b^2$

$$(a - b)^2 = a^2 - 2ab + b^2$$

- The first term in the trinomial is the square of the first term in the binomial.
- The last term in the trinomial is the square of the last term in the binomial.
- The middle term in the trinomial is twice the product of the first and last terms in the binomial.

We can use this pattern to work backwards and factor a perfect square trinomial.

$$\downarrow a^2 + 2ab + b^2 = (a + b)^2 \quad \text{or} \quad \downarrow a^2 - 2ab + b^2 = (a - b)^2$$

Examples:

1) $a^2 + 12a + 36$

$$(a + 6)^2$$

$$\sqrt{a^2} = a$$

$$\sqrt{36} = 6$$

$$2(1)(6) = 12$$

2) $4x^2 + 12x + 9$

$(2x + 3)^2$

$\sqrt{4x^2} = 2x$

$\sqrt{9} = 3$

$2(2)(3) = 12$

3) $25x^2 - 20x + 4$

$(5x - 2)^2$

$\sqrt{25x^2} = 5x$

$\sqrt{4} = 2 \quad 2(5)(2) = 20$

*If we can identify perfect square trinomials, then we don't have to use decomposition to factor them (sometimes the numbers are very large!)

4) $9g^2 + 48g + 64$

$(3g + 8)^2$

$\sqrt{9g^2} = 3g$

$\sqrt{64} = 8$

$2(3)(8) = 48$

Determine if the following are P.S.T.'s:

1) $y^2 + 8y - 16$

no!

2) $4x^2 - 36x + 81$

$\sqrt{81} \quad \sqrt{4}$

$\downarrow \quad \downarrow$

$2(9)(2) = 36$

$(2x - 9)^2$

Factor the following:

1) $a^2 + 14a + 49$

$(a + 7)^2$

2) $49r^2 - 28r + 4$

\uparrow

$(7r - 2)^2$

3) $25n^2 + 30n + 9$

$(5n + 3)^2$

4) $9a^2 - 42a + 49$

$(3a - 7)^2$