

# Lesson 1 Representing Relations and Properties of Functions

## A. Representing Relations

### Vocabulary:

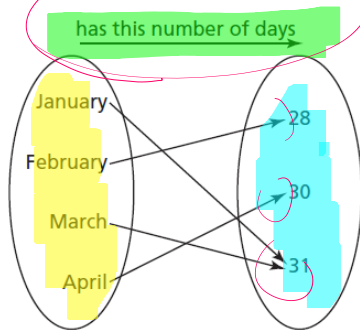
**Set:** a collection of distinct items. i.e.  $\{1,2,3,4,5\}$

**Element:** one item in a set. The order of elements in a set does not matter.

**Relation:** when the elements of one set associate with the elements of another set.

There are 6 ways to represent a relation:

1) Arrow Diagram



$\{\text{JAN., FEB., MAR., APR.}\}$

$\{28, 30, 31\}$

2) Table of Values

Month	Days
January	31
February	28
March	31
April	30

INDEPENDENT VARIABLES

DEPENDENT VARIABLES

3) In Words

The relation shows the association "has the number of" from a set of Months to a set of Days.

4) A set of Ordered Pairs  $(x, y)$

$\{(\text{January}, 31), (\text{February}, 28), (\text{March}, 31), (\text{April}, 30)\}$

5) A graph

6) An equation

## B. Properties of Functions

### Vocabulary:

**Domain:** the set of first elements of a relation (INDEP. VARIABLE, X VALUES)

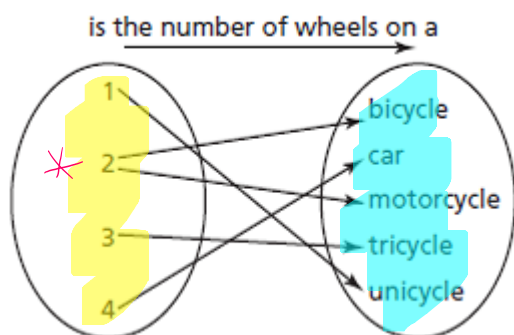
**Range:** the set of related second elements of a relation (DEP. VARIABLE, Y VALUES)

**Function:** a special type of relation where each element in the domain is associated with exactly one element in the range

### Example 1:

Here are some different ways to relate vehicles and the number of wheels each has.

This relation associates a number with a vehicle with that number of wheels.



This diagram does not represent a **function** because there is one element in the first set that associates with two elements in the second set; that is there are two arrows from 2 in the first set.

Ordered Pairs:

$\{(1, \text{unicycle}), (2, \text{bicycle}), (2, \text{motorcycle}), (3, \text{tricycle}), (4, \text{car})\}$

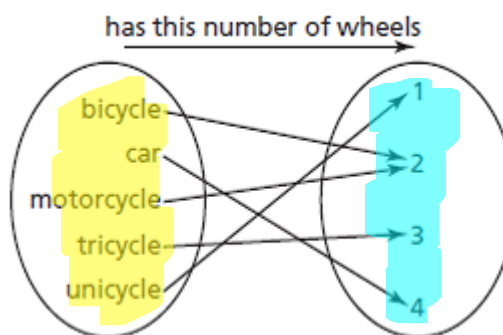
The set of ordered pairs above does not represent a function because two ordered pairs have the same first element.

The **domain** is the set of the first elements:

$\{1, 2, 3, 4\}$

The **range** is the set of associated second elements:  $\{\text{unicycle}, \text{bicycle}, \text{motorcycle}, \text{tricycle}, \text{car}\}$

This relation associates a vehicle with the number of wheels it has.



This diagram represents a **function** because each element in the first set associates with exactly one element in the second set; that is, there is only one arrow from each element in the first set.

Ordered Pairs:

$\{(\text{unicycle}, 1), (\text{bicycle}, 2), (\text{motorcycle}, 2), (\text{tricycle}, 3), (\text{car}, 4)\}$

The set of ordered pairs above represents a function because the ordered pairs have different first elements.

The **domain** is the set of first elements:

$\{\text{unicycle}, \text{bicycle}, \text{motorcycle}, \text{tricycle}, \text{car}\}$

The **range** is the set of associated second elements:  $\{1, 2, 3, 4\}$

## How do you determine if a relation is a function?

DO ANY VALUES IN THE DOMAIN REPEAT?

- 1) Check to see if any ordered pairs have the **same first element**. If not, then for every first element there is exactly one second element, therefore the relation is a function.
- 2) Check to see if any element in the first set associates with more than one element in the second set. If it does, the relation is **NOT** a function.

IF NOT, IT IS A FUNCTION (FX'N)

You can look closely at the ordered pairs, in a table or in an arrow diagram to see if the above are true.

**Example 2:** Is  $\{(6, -3), (4, 1), (7, -2), (-3, 1)\}$  a function? Identify the domain and range.

The domain is:  $\{6, 4, 7, -3\}$  NO Repeat  
 The range is:  $\{-3, -2, 1, 1\} \rightarrow \{-3, -2, 1\}$

The domain is paired with exactly one element of the range, the relation IS a function.

The INVERSE function is NOT a function since 1 is paired with more than one element of the range.

With an **inverse function** we REVERSE the order of the order pairs.

**Example 3:** For each relation below:

- Determine whether the relation is a function. Justify your answer.
- Identify the domain and range of each relation.

a) A relation that associates a number with a prime factor of the number:

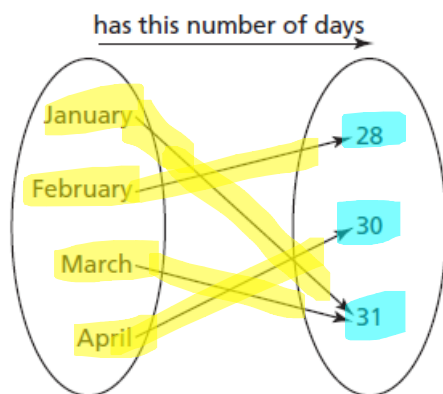
$\{(4, 2), (6, 2), (6, 3), (8, 2), (9, 3)\}$

NOT a function (6 repeats)

$D: \{4, 6, 8, 9\}$

$R: \{2, 3\}$

b)



IS A FUNCTION (ONLY ONE OUTPUT FOR EACH INPUT)

$D: \{JAN, FEB, MAR, APR\}$

$R: \{28, 30, 31\}$

## Dependent and Independent Variables

**Independent variable:**  $(x\text{-axis})$  VARIABLE NOT CHANGED BY THE OTHER VARIABLE.

**Dependent variable:**  $(y\text{-axis})$  VARIABLE THAT DEPENDS ON OTHER FACTORS.

In the workplace, a person's gross pay,  $P$  dollars, often depends on the number of hours worked,  $h$ .

So, we say  $P$  is the **dependent variable**. Since the number of hours worked,  $h$ , does not depend on the gross pay,  $P$ , we say that  $h$  is the **independent variable**.

independent variable $\rightarrow$	$x$ Hours Worked, $h$	$y$ Gross Pay, $P$ (\$)	$\leftarrow$ dependent variable
domain	1	12	range
	2	24	
	3	36	
	4	48	
	5	60	

A table of values usually represents a sample of the ordered pairs in a relation.

The values of the independent variable are listed in the first column of a table of values. These elements belong to the domain.

The values of the dependent variable are listed in the second column of a table of values. These elements belong to the range.

We can think of a function as an input/output machine. The input can be any number in the domain, and the output is dependent on the input.

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