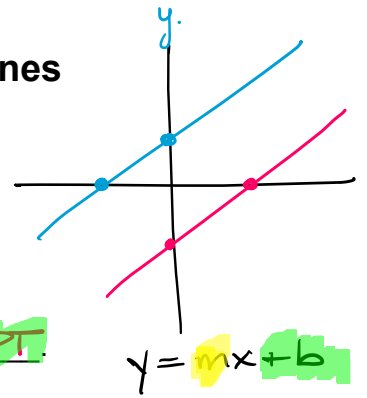


Lesson 2: Slopes of Parallel and Perpendicular Lines

A. Parallel Lines

- 1) Never cross
- 2) Have the **same SLOPE** (m).
- 3) Have **different X-INTERCEPT** and **Y-INTERCEPT** (b).

How STEEP THE LINE IS.



Examples:

1) Which of the following is/are parallel to $y = 2x + 3$? $m = 2$ $b = +3$

a) $y = 3x - 2$

$m = 3$ $b = -2$

NOT PARALLEL BECAUSE SLOPES ARE DIFFERENT.

b) $y - 1 = 2(x + 3)$

$y - 1 = 2x + 6$
 $y = 2x + 7$

$m = 2$ ✓ SAME
 $b = +7$ ✓ DIFFERENT

PARALLEL

c) $2x - y + 3 = 0$

$2x + 3 = y$

$m = 2$ ✓ SAME
 $b = +3$ ✗ SHOULD BE DIFFERENT

SAME LINE NOT PARALLEL

2) Line EF passes through E(-3,-2) and F(-1,6). Line CD passes through C(-1,-3) and D(1,7). Line AB passes through A(-3,7) and B(-5,-2). Sketch the lines. Are they parallel? Justify your answer.

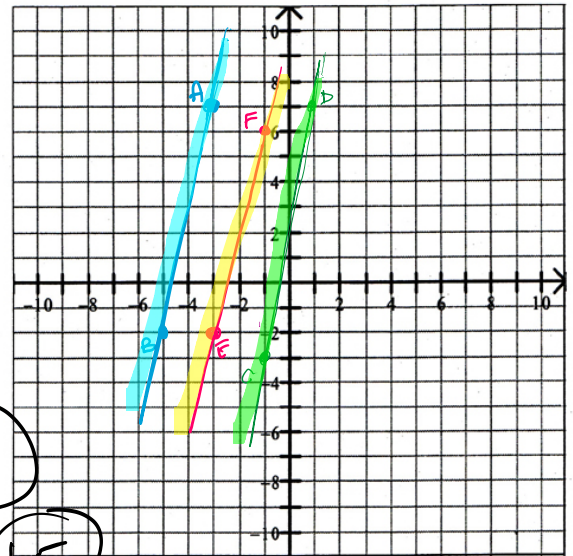
$m = \frac{y_2 - y_1}{x_2 - x_1}$

E (-3, -2)
F (-1, 6)
C (-1, -3)
D (1, 7)
A (-3, 7)
B (-5, -2)

$m_{EF} = \frac{6 - (-2)}{-1 - (-3)} = \frac{8}{2} = 4$

$m_{CD} = \frac{7 - (-3)}{1 - (-1)} = \frac{10}{2} = 5$

$m_{AB} = \frac{-2 - 7}{-5 - (-3)} = \frac{-9}{-2} = 4.5$



NO PARALLEL LINES

3) The coordinates of the endpoints of segments are given as A(3,-1), B(7,1) and C(5,6), D(13,10). Are the two line segments parallel? $x_1 y_1$ $x_2 y_2$

$SLOPE(m) = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \frac{1 - (-1)}{7 - 3} = \frac{2}{4} = \frac{1}{2}$

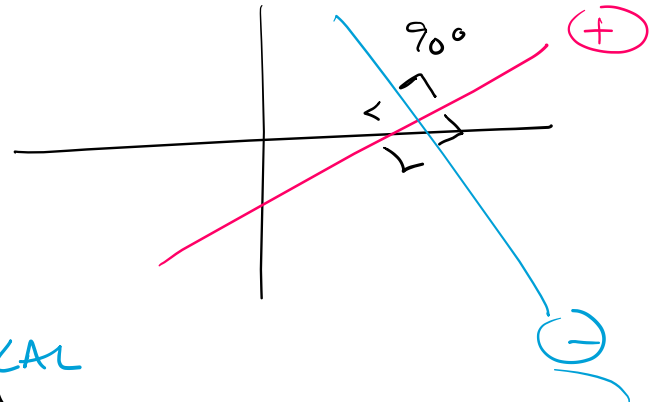
$m_{CD} = \frac{10 - 6}{13 - 5} = \frac{4}{8} = \frac{1}{2}$

PARALLEL

(b/c they have the same slope)

B. Perpendicular Lines

- 1) Cross at a 90° angle
- 2) Slopes:
 - (i) have opposite direction (+/-)
 - (ii) are NEGATIVE RECIPROCAL



Examples:

OPPOSITE FLIPPED

- 1) Which of the following are perpendicular to $y = \frac{2}{3}x + 1$? $m = \frac{2}{3}$
- a) $y = -\frac{3}{2}x + 3$ b) $y - 1 = \frac{-2}{3}(x + 3)$ c) $3x + 2y - 6 = 0$

$$m = -\frac{3}{2}$$

YES, PERPENDICULAR

$$-\frac{3}{2} \perp \frac{2}{3}$$

$$y - 1 = -\frac{2}{3}x - 2$$

$$y = -\frac{2}{3}x - 1$$

NOT PERPENDICULAR

$$3x + 2y = 6$$

$$-3x \quad -3x$$

$$2y = -3x + 6$$

$$y = -\frac{3}{2}x + 3$$

YES!

$y = mx + b$
CHECKING FOR NEG. RECIP.

negative reciprocal

- 2) a) Determine the slope of a line that is perpendicular to the line through $G(-2, 3)$ and $H(1, -2)$.

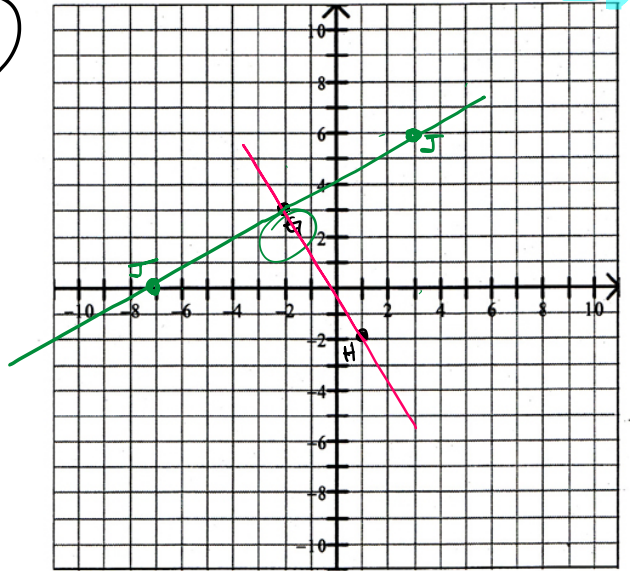
$$m_{GH} = \frac{-2 - 3}{1 - (-2)} = \frac{-5}{3}$$

$$\frac{1}{m_{GH}} = +\frac{3}{5}$$

- b) Determine the coordinates of J so that line GJ is perpendicular to line GH.

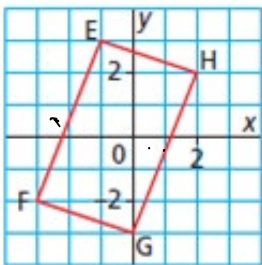
$J(-7, 0)$ or $(3, 6)$

USE $\perp m = \frac{3}{5}$ up 3 right 5
or down 3 left 5



- 3) EFGH is a parallelogram. Is it a rectangle? Justify your answer.

- E(-1, 3)
F(-3, -2)
G(0, -3)
H(2, 2)



$$m_{EF} = \frac{-2 - 3}{-3 - (-1)} = \frac{-5}{-2} = \frac{5}{2}$$

$$m_{FG} = \frac{-3 - (-2)}{0 - (-3)} = \frac{-1}{3}$$

$$m_{GH} = \frac{2 - (-3)}{2 - 0} = \frac{5}{2}$$

$$m_{EH} = \frac{2 - 3}{2 - (-1)} = \frac{-1}{3}$$

NOT NEG. RECIP. SO NOT A RECTANGLE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- 4) The coordinates of the endpoints of segments are given as A(24,-1), B(12,5) and C(-4,1), D(2,13). Are the two line segments parallel, perpendicular, or neither?

$$\begin{array}{l} A(24, -1) \\ B(12, 5) \\ \hline C(-4, 1) \\ D(2, 13) \end{array}$$

$$m_{AB} = \frac{5 - (-1)}{12 - 24} = \frac{6}{-12} = -\frac{1}{2}$$

$$m_{CD} = \frac{13 - 1}{2 - (-4)} = \frac{12}{6} = 2 \text{ or } \frac{2}{1}$$

$m_{AB} \perp m_{CD}$ MEANING THE LINES ARE PERPENDICULAR

- 5) Line ST passes through S(-2,7) and T(2,-5). Line UV passes through U(-2,3) and V(7,6).

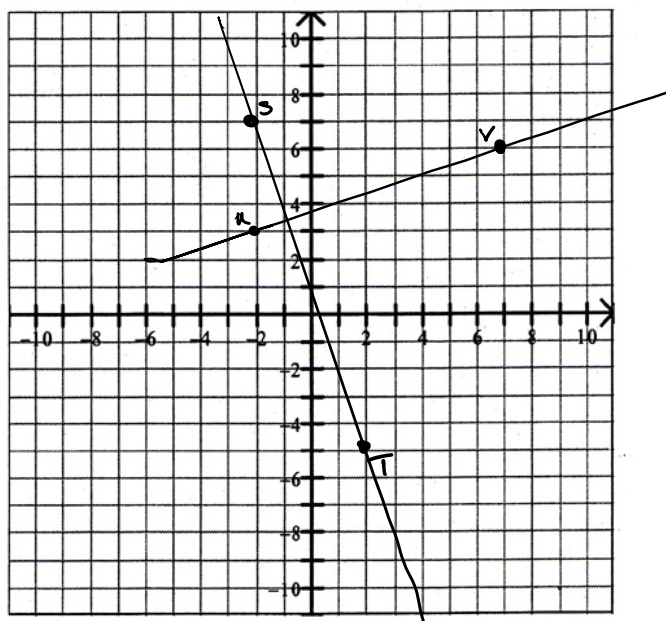
- a) Are these two lines parallel, perpendicular, or neither? Justify your answer.

$$m_{ST} = \frac{-5 - 7}{2 - (-2)} = \frac{-12}{4} = -3$$

$$m_{UV} = \frac{6 - 3}{7 - (-2)} = \frac{3}{9} = \frac{1}{3}$$

PERPENDICULAR
B/C $m_{ST} \perp m_{UV}$
NEQ. RECIPR.

- b) Sketch the lines to verify your answer to part a).

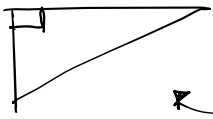


$$S(-2, 7)$$

$$T(2, -5)$$

$$U(-2, 3)$$

$$V(7, 6)$$



6) Triangle ABC has vertices A(-1,-2), B(-4,0), and C(-2,3). Verify that it is a right triangle.

MUST HAVE
90° SO
WILL HAVE
NEG. RECIP.

1. Draw a graph/picture.

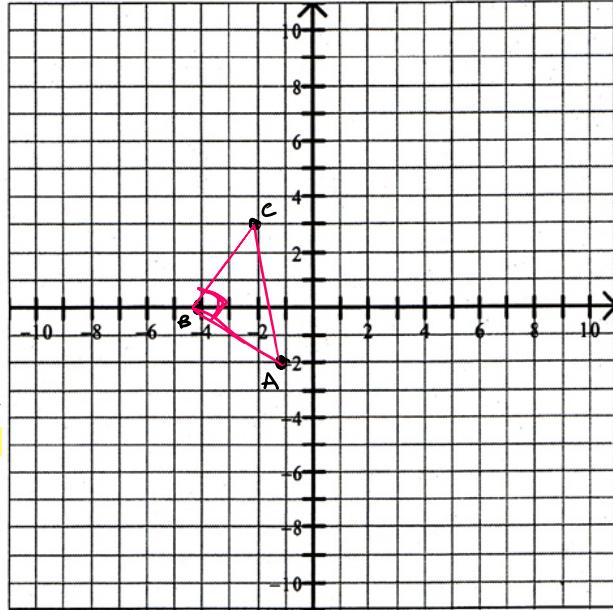
2. Find the slopes.

BASED ON GRAPH
 m_{AB} SHOULD BE
A NEG. RECIP. OF
 m_{BC} .

$$m_{AB} = \frac{0 - (-2)}{-4 - (-1)} = \frac{2}{-3}$$

$$m_{BC} = \frac{3 - 0}{-2 - (-4)} = \frac{3}{2}$$

3. Check for perpendiculars.



$\triangle ABC$ IS A RIGHT TRIANGLE BECAUSE

$$m_{AB} \perp m_{BC}$$