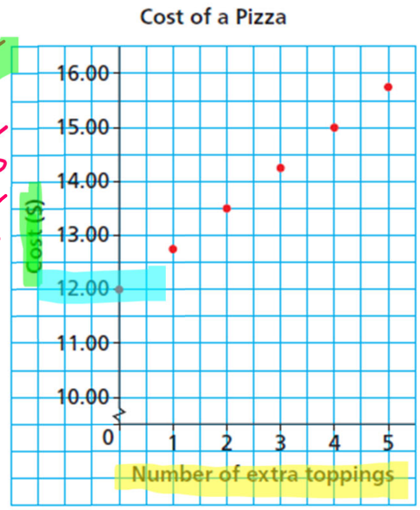
Lesson 5: Properties of Linear Relations

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

Number of Extra Toppings (independent, x)	Cost ($) (dependent, y)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

+1 +1 +1

+0.75 +0.75 +0.75



$m = \frac{\text{rise}}{\text{run}} = \frac{0.75}{1} = 0.75$

a) What patterns do you see in the table?

AS THE # OF TOPPINGS INCREASE BY ONE, THE COST INCREASES BY $0.75 (75¢)

b) Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

m = SLOPE
b = Y-INTERCEPT
→ EQUATION ($y = mx + b$)

$y = mx + b$

$C = 0.75T + 12.00$

c) How are the patterns in the table shown in the graph?

SLOPE DOESN'T CHANGE = CONSTANT CHANGE (ALWAYS 0.75)

d) How can you tell from the table that the graph represents a **linear relation**?

CONSTANT CHANGE ⇒ RESULTS IN A STRAIGHT LINE GRAPH

Linear Relations

The cost for a car rental is $60, plus $20 for every 100 km driven. The independent (x) variable is the distance driven and the dependent (y) variable is the cost.

There are several ways to identify that this is a linear relation.

A. Table of Values

Distance (km) (indep., x)	Cost ($) (dependent, y)
0	60
100	80
200	100
300	120
400	140

+100 +100 +100 +100 — CONSTANT

+20 +20 +20 +20 — CONSTANT

* BECAUSE BOTH THE INDEP. & DEP. VARIABLES CHANGE BY THE SAME AMOUNT (REMAIN CONSTANT), THIS IS A LINEAR RELATION

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

B. A set of ordered pairs

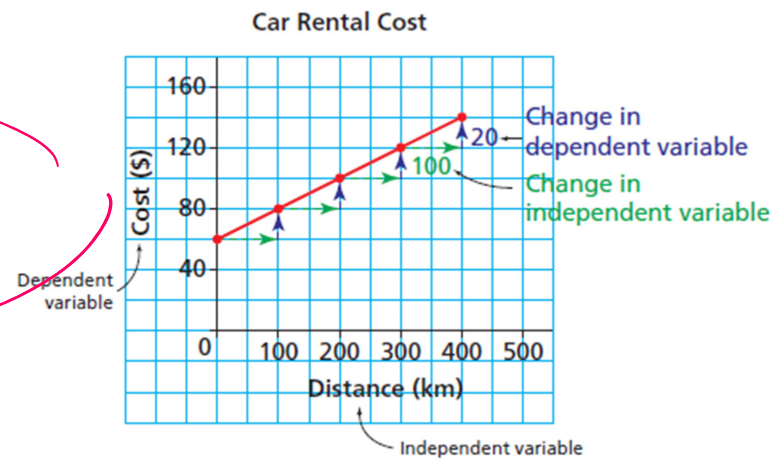
+100 +100 +100 +100

{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}

+20 +20 +20 +20

C. A graph

The graph of a linear relation is a straight line.



By analyzing a table, set of ordered pairs or graph, we can determine the RATE OF CHANGE of the linear relation. The ratio can be set up as follows:
(slope)

$$m = \frac{\text{change in dependent variable } (y)}{\text{change in independent variable } (x)} = \frac{\text{RISE}}{\text{RUN}} = \frac{\$20}{100\text{ km}} = \$0.20/\text{km}$$

The rate of change (slope) can also be determined from the equation of the linear function:

$$C = 0.20d + 60$$

C – dependent
d – independent
m – rate of change (slope)
b – y-intercept

Examples:

1) Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, n, and time, t minutes.

t	n
0	1
20	2
40	4
60	8
80	16
100	32

+20

+1 +2 +4

NOT CONSTANT SO NOT LINEAR

b) The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars

A	T
60	3
120	6
180	9
240	12
300	15

+60 +60 +60

+3 +3

CHANGE IS CONSTANT FOR BOTH VARIABLES SO LINEAR

2) Graph these relations. Are they linear? How do you know?

a) $y = 2x - 1$

x	y
0	-1
2	3
4	7

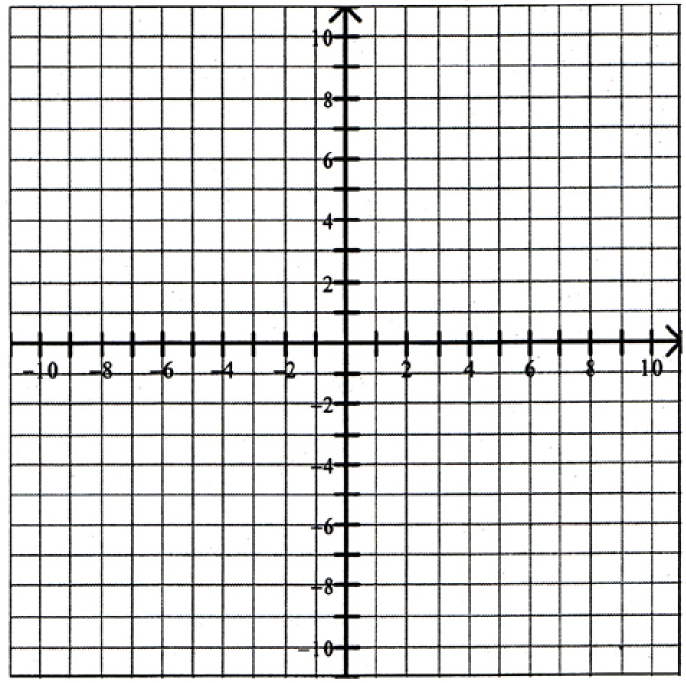
+2 +4

CONSTANT CHANGE SO, LINEAR!

$y = 2(0) - 1 = 0 - 1 = -1$

$y = 2(2) - 1 = 4 - 1 = 3$

$y = 2(4) - 1 = 8 - 1 = 7$



LINEAR B/C STRAIGHT LINE

b) $y = -x^2 + 2$

NOT CONSTANT SO NOT LINEAR

x	y
0	2
1	1
2	-2

+1 +1 -1 -3

$y = -0^2 + 2$

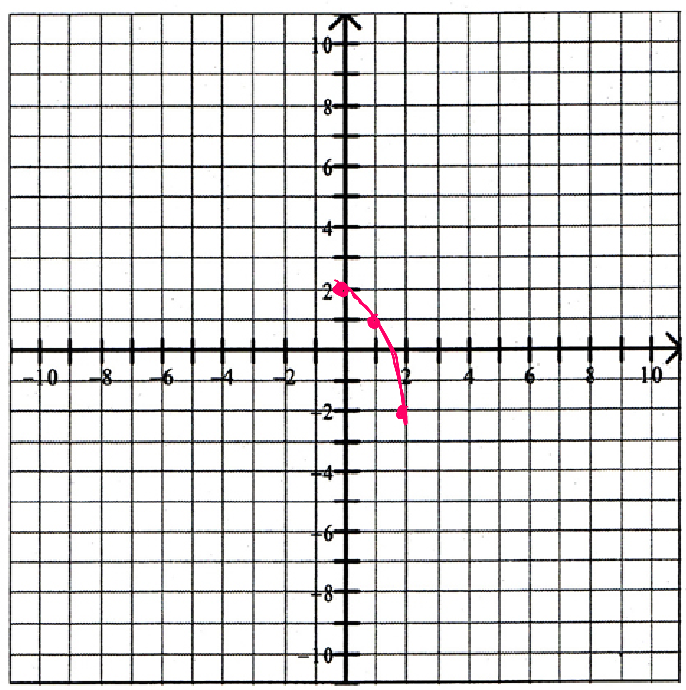
$= 0 + 2$

$y = -(1)^2 + 2$

$= -1 + 2$

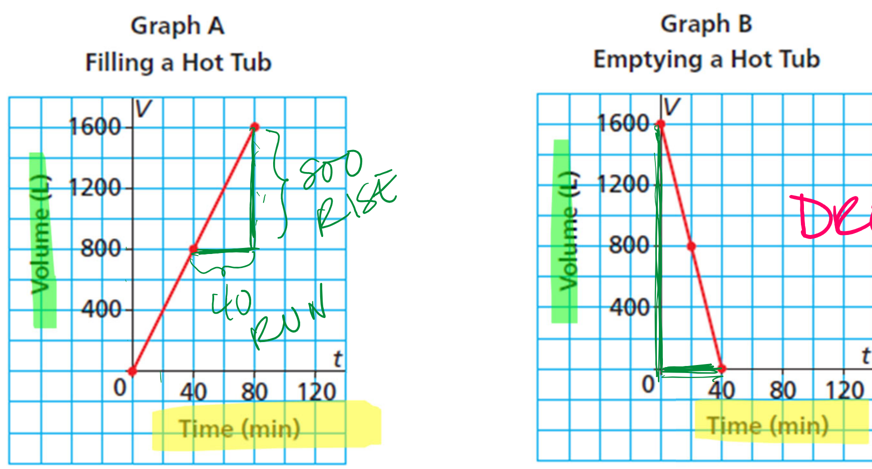
$y = -(2)^2 + 2$

$= -4 + 2$



NOT LINEAR B/C NOT A STRAIGHT LINE

3) A hot tub contains 1000 L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub emptied at a constant rate.



DROPPING! SO RATE OF CHANGE IS NEGATIVE (−)

a) Identify the dependent and independent variables.

volume (L) time (min)

b) Determine the rate of change of each relation, and then describe what it represents.

(= SLOPE)

$$\text{RATE OF CHANGE } (m) = \frac{\text{CHANGE DEP. } (y)}{\text{CHANGE INDEP. } (x)} = \frac{\text{rise}}{\text{run}}$$

GRAPH A

$$m = \frac{800\text{ L}}{40\text{ min}} = 20\ \frac{\text{L}}{\text{min}}$$

FILLING AT A RATE OF 20 LITRES EVERY MINUTE.

GRAPH B

$$m = \frac{-1600\text{ L}}{40\text{ min}} = -40\ \frac{\text{L}}{\text{min}}$$

LOSING 40 LITRES OF WATER EVERY MINUTE.

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