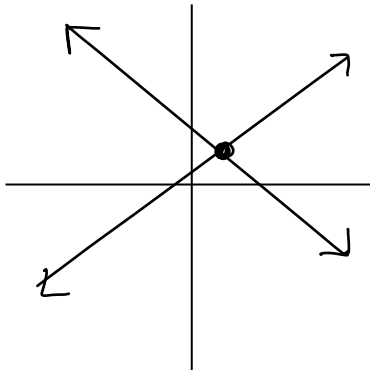


Lesson 5 – Sec. 7.6: Properties of Systems of Linear Equations

When you attempt to solve a linear system of two equations in two variables, there are only **three** different possibilities. You can determine the number of solutions using different methods.

Possible Solutions for a Linear System

1) Intersecting Lines

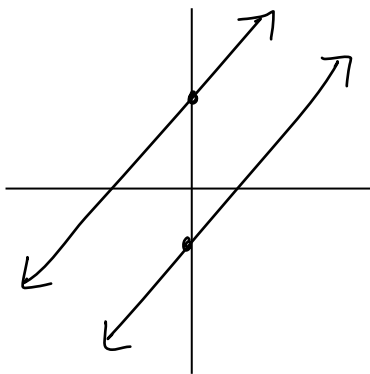


1 solution $P(x, y)$

The lines ($y = mx + b$)

- different slopes
- different y-int.

2) Parallel Lines



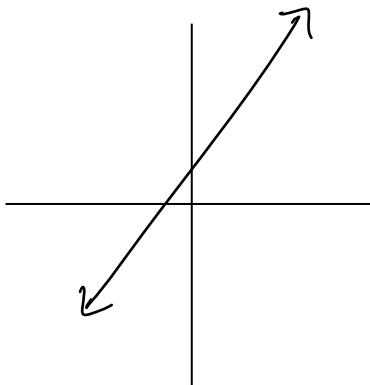
No solution

The lines

- same slope
- different y-int

* inconsistent system

3) Coincident Lines



Infinite solutions

- same line
- same slope
- same y-int.

Intersecting } consistent systems (have solution)
Coincident } Marsh

Example 1: Determine the number of solutions of each linear system.

a) $x + y = -2$ * get into $y = mx + b$

$$\begin{array}{r} -2x - 2y = 4 \\ +2x \quad +2x \end{array}$$

$$\frac{-2y}{-2} = \frac{2x+4}{-2} \quad \frac{4}{-2}$$

$$y = -x - 2$$

$$\begin{array}{r} x + y = -2 \\ -x \quad -x \end{array}$$

$$y = -x - 2$$

same line

Coincident lines w/ infinite solutions

b) $2x + 6y = -10$

$$\begin{array}{r} -2x - y = -1 \\ +2x \quad +2x \end{array}$$

$$\frac{-y}{-1} = \frac{2x-1}{-1} \quad \frac{-1}{-1}$$

$$y = -2x + 1$$

$$\begin{array}{r} 2x + 6y = -10 \\ -2x \quad -2x \end{array}$$

$$\frac{6y}{6} = \frac{-2x-10}{6} \quad \frac{-10}{6}$$

$$y = -\frac{x}{3} - \frac{5}{3}$$

Intersecting Lines \rightarrow 1 solution

c) $3x + y = -1$

$$\begin{array}{r} -6x - 2y = 12 \\ +6x \quad +6x \end{array}$$

$$\frac{-2y}{-2} = \frac{6x+12}{-2} \quad \frac{12}{-2}$$

$$y = -3x - 6$$

$$\begin{array}{r} 3x + y = -1 \\ -3x \quad -3x \end{array}$$

$$y = -3x - 1$$

same slope

Parallel, no solution.

$$y = 2x + 4 \quad m = 2$$
$$b = 4$$

Example 2: Given the equation $-2x + y = 4$, write another linear equation that will form a linear system with:

- a) Exactly one solution \rightarrow different m, b
- b) No solution
- c) Infinite solutions

a) choices are endless

$$y = 3x - 1$$
$$y = 1000000x + 9001$$

b) same slope

$$y = 2x + 9$$

or

$$y = 2x - 5$$

c) same line $y = 2x + 4.$